The application of linear programming to management accounting

Solutions to Chapter 26 questions

Question 26.16

(a)

<table>
<thead>
<tr>
<th></th>
<th>M</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contribution per unit</td>
<td>£96</td>
<td>£110</td>
</tr>
<tr>
<td>Litres of material P required</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>Contribution per litre of material P</td>
<td>£12</td>
<td>£11</td>
</tr>
<tr>
<td>Ranking</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Production/sales (units)</td>
<td>1000</td>
<td>2325a</td>
</tr>
</tbody>
</table>

Note:

a 31 250 litres of P less (1000 × 8) for M = 23 250 litres for F giving a total production of 2325 units (23 250 litres/10)

(b)

<table>
<thead>
<tr>
<th></th>
<th>M (£000)</th>
<th>F (£000)</th>
<th>Total (£000)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales</td>
<td>200</td>
<td>488.250</td>
<td>688.250</td>
</tr>
<tr>
<td>Variable costs:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Material P</td>
<td>20</td>
<td>58.125</td>
<td>78.125</td>
</tr>
<tr>
<td>Material Q</td>
<td>40</td>
<td>46.500</td>
<td>86.500</td>
</tr>
<tr>
<td>Direct labour</td>
<td>28</td>
<td>81.375</td>
<td>109.375</td>
</tr>
<tr>
<td>Overhead</td>
<td>16</td>
<td>46.500</td>
<td>62.500</td>
</tr>
<tr>
<td></td>
<td>104</td>
<td>232.500</td>
<td>336.500</td>
</tr>
<tr>
<td>Contribution</td>
<td>96</td>
<td>255.750</td>
<td>351.750</td>
</tr>
<tr>
<td>Fixed costs (£150 000 + £57 750)</td>
<td></td>
<td></td>
<td>207.750</td>
</tr>
<tr>
<td>Profit</td>
<td></td>
<td></td>
<td>144.000</td>
</tr>
</tbody>
</table>

(c) Maximize Z = 96M + 110F (product contributions) subject to:

8M + 10F ≤ 31 250 (material P constraint)
10M + 5F ≤ 20 000 (material Q constraint)
4M + 5F ≤ 17 500 (direct labour constraint)
M ≤ 1 000 (maximum demand for M)
F ≤ 3 000 (maximum demand for F)

The above constraints are plotted on the graph shown in Figure Q26.16 as follows:

Material P; Line from M = 3906.25, F = 0 to F = 3125, M = 0
Material Q; Line from M = 2000, F = 0 to F = 4000, M = 0
Direct labour; Line from M = 4375, F = 0 to F = 3500, M = 0
Sales demand of M; Line from M = 1000
Sales demand of F; Line from F = 3000

The optimal solution occurs where the lines in Figure 26.16 intersect for material P and Q constraints. The point can be determined from the graph or mathematically as follows:

8M + 10F = 31 250 (material P constraint)
10M + 5F = 20 000 (material Q constraint)

multiplying the first equation by 1 and the second equation by 2:
8M + 10F = 31 250
20M + 10F = 40 000
subtracting –12M = – 8750
M = 729.166

Substituting for M in the first equation:
8(729.166) + 10F = 31 250
F = 2541.667

(d)

Contribution: (729 units of M at £96) 69 984
(2542 units of F at £110) 279 620

Less fixed costs 349 604
Profit 141 854
Moving from the solution in (c) where the lines intersect as a result of obtaining an additional litre of material Q gives the following revised equations:

\[ 8M + 10F = 31250 \] (material P constraint)
\[ 10M + 5F = 20001 \] (material Q constraint)

The values of M and F when the above equations are solved are 729.333 and 2541.533. Therefore, M is increased by 0.167 units and F is reduced by 0.134 units giving an additional total contribution of £1.292 \([0.167 \times £96) – (0.134 \times £110)]\) per additional litre of Q. Therefore the shadow price of Q is £1.292 per litre.

(e) See Chapter 26 for an explanation of shadow prices.

(f) Other factors to be taken into account include the impact of failing to meet the demand for product M, the need to examine methods of removing the constraints by sourcing different markets for the materials and the possibility of sub-contracting to meet the unfulfilled demand.

**Question 26.17**

(a) Let \(X\) = number of units of XL produced each week
    \(Y\) = number of units of YM produced each week
    \(Z\) = total contribution

The linear programming model is:

Maximize \(Z = 40X + 30Y\) (product contributions) subject to

\[ 4X + 4Y \leq 120 \] (materials constraint)
\[ 4X + 2Y \leq 100 \] (labour constraint)
\[ X + 2Y \leq 50 \] (plating constraint)
\[ X, Y \geq 0 \]

The above constraints are plotted on Figure Q26.17. The optimum output is at point C on the graph, indicating that 20 units of XL and 10 units of YM should be produced. The optimum output can be determined exactly by solving the simultaneous equations for the constraints that intersect at point C:

\[ 4X + 4Y = 120 \]
\[ 4X + 2Y = 100 \]

Subtracting
\[ 2Y = 20 \]
\[ Y = 10 \]
Substituting for $Y$:

$$4X + 40 = 120$$
$$X = 20$$

The maximum weekly profit is:

$$(20 \times £40) + (10 \times £30) - £700 \text{ fixed costs} = £400$$

(b) The present objective function is $40X + 30Y$ and the gradient of this line is $−40/30$. If the selling price of YM were increased, the contribution of YM would increase and the gradient of the line ($−40/30$) would decrease. The current optimal point is C because the gradient of the objective function line is greater than the gradient of the line for the constraint of materials (the line on which the optimal point C falls). If the gradient of the objective function line were equal to the gradient of the line for the materials constraint, the optimal solution would be any point on FC. The gradient for the materials constraint line is $−1$. If the gradient for the objective function line were less than $−1$, the optimal solution would change from point C to point B. The gradient of the line for the current objective function of $40X + 30Y$ will be greater than $−1$ as long as the contribution from YM is less than £40. If the contribution from YM is £40 or more, the optimum solution will change. Therefore the maximum selling price for YM is £190 (£150 variable cost + £40 contribution).

(c) If plating time can be sold for £16 per hour then any hour devoted to XLs and YMs loses £16 sales revenue. The relevant cost per plating hour is now £16 opportunity cost. The contributions used in the objective function should be changed to reflect this opportunity cost. The contribution should be reduced by £4 (1 hour at £16 − £12) for XL and by £8 (2 hours at £16 − £12) for YL. The revised objective function is:

$$Z = 36X + 22Y$$

(d) The scarce resources are materials and labour. This is because these two constraints intersect at the optimal point C. Plating is not a scarce resource, and the shadow price is zero.

If we obtain an additional unit of materials the revised constraints will be:

$$4X + 4Y = 121 \text{ (materials)}$$
$$4X + 2Y = 100 \text{ (labour)}$$

The values of $X$ and $Y$ when the above equations are solved at 10.5 for $Y$ and 19.75 for $X$. Therefore YM is increased by 0.5 units and XL is reduced by 0.25 units and the change in contribution will be as follows:

| (£) | Increase in contribution of YM ($0.5 \times £30$) | 15 |
| Decrease in contribution of XL ($0.25 \times £40$) | 10 |
| Increase in contribution (shadow price) | 5 |

If we obtain one additional labour hour, the revised constraints will be:

$$4X + 4Y = 120 \text{ (materials)}$$
$$4X + 2Y = 101 \text{ (labour)}$$

The values of $X$ and $Y$ when the above equations are solved are 9.5 for $Y$ and 20.5 for $X$. Therefore XL is increased by 0.5 units and YM is reduced by 0.5 units, and the change in contribution will be as follows:

| (£) | Increase in contribution from XL ($0.5 \times £40$) | 20 |
| Decrease in contribution from YM ($0.5 \times £30$) | 15 |
| Increase in contribution (shadow price) | 5 |
The relevant cost of resources used in producing ZN consists of the acquisition cost plus the shadow price (opportunity cost). The relevant cost calculation is:

(£)

<table>
<thead>
<tr>
<th>Description</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material A [5 kg at (£10 + £5)]</td>
<td>75</td>
</tr>
<tr>
<td>Labour [5 hours at (£8 + £5)]</td>
<td>65</td>
</tr>
<tr>
<td>Plating (1 hour at £12)</td>
<td>12</td>
</tr>
<tr>
<td>Other variable costs</td>
<td>90</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>242</strong></td>
</tr>
</tbody>
</table>

The selling price is less than the relevant cost. Therefore product ZN is not a profitable addition to the product range.

(e) The shadow price of labour is £5 per hour. Therefore the company should be prepared to pay up to £5 in excess of the current rate of £8 in order to remove the constraint. An overtime payment involves an extra £4 per hour, and therefore overtime working is worthwhile.

Increasing direct labour hours will result in the labour constraint shifting to the right. However, when the labour constraint line reaches point E, further increases in labour will not enable output to be expanded (this is because other constraints will be binding). The new optimal product mix will be at point E, with an output of 30 units of XL and zero of YM. This product mix requires 120 hours (30 × 4 hrs). Therefore 120 labour hours will be worked each week. Note that profit will increase by £20 (20 × (£5 – £4)).

(f) The limitations are as follows:

(i) It is assumed that the objective function and the constraints are linear functions of the two variables. In practice, stepped fixed costs might exist or resources might not be used at a constant rate throughout the entire output range. Selling prices might have to be reduced to increase sales volume.

(ii) Constraints are unlikely to be completely fixed and as precise as implied in the mathematical model. Some constraints can be removed at an additional cost.

(iii) The output of the model is dependent on the accuracy of the estimates used. In practice, it is difficult to segregate costs accurately into their fixed and variable elements.

(iv) Divisibility of output is not realistic in practice (fractions of products cannot be produced). This problem can be overcome by the use of integer programming.

(v) The graphical approach requires that only two variables (products) be considered. If several products compete for scarce resources, it will be necessary to use the Simplex method.

(vi) Qualitative factors are not considered. For example, if overtime is paid, the optimum solution is to produce zero of product YM. This will result in the demand from regular customers for YM (who might also buy XM) not being met. This harmful effect on customer goodwill is not reflected in the model.

Question 26.18

(a) The calculation of the contributions for each product is:

<table>
<thead>
<tr>
<th></th>
<th>X1 (£)</th>
<th>X2 (£)</th>
<th>X3 (£)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Selling price</td>
<td>83</td>
<td>81</td>
<td>81</td>
</tr>
<tr>
<td>Materials</td>
<td>(51)</td>
<td>(45)</td>
<td>(54)</td>
</tr>
<tr>
<td>Manufacturing costs</td>
<td>(11)</td>
<td>(11)</td>
<td>(11)</td>
</tr>
<tr>
<td>Contribution</td>
<td>21</td>
<td>25</td>
<td>16</td>
</tr>
</tbody>
</table>

Notes

*The material cost per tonne for each product is:
\[
X_1 = (0.1 \times £150) + (0.1 \times £60) + (0.2 \times £120) + (0.6 \times £10) = £51 \\
X_2 = (0.1 \times £150) + (0.2 \times £60) + (0.1 \times £120) + (0.6 \times £10) = £45 \\
X_3 = (0.2 \times £150) + (0.1 \times £60) + (0.1 \times £120) + (0.6 \times £10) = £54 \\
\]

It is assumed that manufacturing costs do not include any fixed costs. The initial linear programming model is as follows:

Maximize \[ Z = 21X_1 + 25X_2 + 16X_3 \]
subject to
\[ 0.1X_1 + 0.1X_2 + 0.2X_3 \leq 1200 \text{ (nitrate)} \]
\[ 0.1X_1 + 0.2X_2 + 0.1X_3 \leq 2000 \text{ (phosphate)} \]
\[ 0.2X_1 + 0.1X_2 + 0.1X_3 \leq 2200 \text{ (potash)} \]
\[ X_1, X_2, X_3 \geq 0 \]

(b) The slack variables are introduced to represent the amount of each of the scarce resources unused at the point of optimality. This enables the constraints to be expressed in equalities. The initial Simplex tableau is:

\[
\begin{array}{cccc}
X_1 & X_2 & X_3 & \text{(contribution)} \\
X_4 \text{ (nitrate)} = 1200 & -0.1 & -0.1 & -0.2 \\
X_5 \text{ (phosphate)} = 2000 & -0.1 & -0.2 & 0.1 \\
X_6 \text{ (potash)} = 2200 & -0.2 & -0.1 & -0.1 \\
Z & 21 & 25 & 16 \\
\end{array}
\]

(c) The starting point for the first iteration is to select the product with the highest contribution (that is, \(X_2\)), but production of \(X_2\) is limited because of the input constraints. Nitrate (\(X_4\)) limits us to a maximum production of 12 000 tonnes (1200/0.1), \(X_5\) to a maximum production of 10 000 tonnes (2000/0.2) and \(X_6\) to a maximum production of 22 000 tonnes (2200/0.1). We are therefore restricted to a maximum production of 10 000 tonnes of product \(X_2\) because of the \(X_5\) constraint. The procedure which we should follow is to rearrange the equation that results in the constraint (that is, \(X_5\)) in terms of the product we have chosen to make (that is, \(X_2\)). Therefore the \(X_5\) equation is re-expressed in terms of \(X_2\), and \(X_5\) will be replaced in the second iteration by \(X_2\). (Refer to ‘Choosing the product’ in Chapter 26 if you are unsure of this procedure.) Thus \(X_2\) is the entering variable and \(X_5\) is the leaving variable.

(d) Following the procedure outlined in Chapter 26, the final tableau given in the question can be reproduced as follow:

\[
\begin{array}{cccccc}
\text{Quantity} & X_3 & X_4 & X_5 \\
X_1 & 4000 & -3 & -20 & +10 \\
X_2 & 8000 & +1 & +10 & -10 \\
X_6 & 600 & +0.4 & +3 & -1 \\
Z & 284000 & -22 & -170 & -40 \\
\end{array}
\]

In Chapter 26 the approach adopted was to formulate the first tableau with positive contribution signs and negative signs for the slack variable equations. The optimal solution occurs when the signs in the contribution row are all negative. The opposite procedure has been applied with the tableau presented in the question. Therefore the signs have been reversed in the above tableau to ensure it is in the same format as that presented in Chapter 26. Note that when an entry of 1 is shown in a row or column for a particular product or slack variable then the entry does not appear in the above tableau. For example, \(X_1\) has an entry of 1 for the \(X_1\) row and \(X_1\) column. These cancel out and the entry is not made in the above tableau. Similarly, an entry of 1 is omitted in respect of \(X_2\) and \(X_6\).

The optimum solution is to produce 4000 tonnes of \(X_1\), 8000 tonnes of \(X_2\) and zero \(X_3\) each month. This gives a monthly contribution of £284 000, uses all the nitrate (\(X_4\)) and phosphate (\(X_5\)), but leaves 600 tonnes of potash (\(X_6\)) unused.
The opportunity costs of the scarce resources are:

<table>
<thead>
<tr>
<th>Resource</th>
<th>Cost per tonne</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nitrate  (X4)</td>
<td>£170</td>
</tr>
<tr>
<td>Phosphate (X5)</td>
<td>£40</td>
</tr>
</tbody>
</table>

If we can obtain an additional tonne of nitrate then output of X1 should be increased by 20 tonnes and output of X2 should be reduced by 10 tonnes.

Note that we reverse the signs when additional resources are obtained. The effect of this substitution process on each of the resources and contribution is as follows:

<table>
<thead>
<tr>
<th>Effect on Resources</th>
<th>Contribution (£)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Increase X1 by 20 tonnes</td>
<td>+420</td>
</tr>
<tr>
<td>Reduce X2 by 10 tonnes</td>
<td>-250</td>
</tr>
<tr>
<td>Net effect</td>
<td>170</td>
</tr>
</tbody>
</table>

The net effect agrees with the X4 column in the final tableau. That is, the substitution process will use up exactly the one additional tonne of nitrate, 3 tonnes of unused resources of potash and increase contribution by £170.

To sell one unit of X3, we obtain the resources by reducing the output of X1 by 3 tonnes and increasing the output of X2 by 1 tonne. (Note the signs are not reversed, because we are not obtaining additional scarce resources.) The effect of this substitution process is to reduce contribution by £22 for each tonne of X3 produced. The calculation is as follows:

Increase X3 by 1 tonne = +£16 contribution
Increase X2 by 1 tonne = +£25 contribution
Reduce X1 by 3 tonnes = -£63
Loss of contribution = -£22

(e) (i) Using the substitution process outlined in (d), the new values if 100 extra tonnes of nitrate are obtained will be:

\[
X1 \text{ 4000 } + \text{ (20 } \times \text{ 100)} = 6000
\]
\[
X2 \text{ 8000 } - \text{ (10 } \times \text{ 100)} = 7000
\]
\[
X6 \text{ 600 } - \text{ (3 } \times \text{ 100)} = 300
\]

Contribution \(284 \text{ 000 } + \text{ (£170 } \times \text{ 100)} = £301 \text{ 000}\)

Hence the new optimal solution is to make 6000 tonnes of X1 and 7000 tonnes of X2 per month, and this output will yield a contribution of £301 000.

(ii) Using the substitution process outlined in (d), the new values if 200 tonnes per month of X3 are supplied will be:

\[
X1 \text{ 4000 } - \text{ (3 } \times \text{ 200)} = 3400
\]
\[
X2 \text{ 8000 } + \text{ (1 } \times \text{ 200)} = 8200
\]
\[
X6 \text{ 600 } + \text{ (0.4 } \times \text{ 200)} = 680
\]
\[
X3 \text{ 0 } + \text{ (1 } \times \text{ 200)} = 200
\]

Contribution £284 000 - (£22 x 200) = 279 600

Hence the new optimal solution is to produce 3400 tonnes of X1, 8200 tonnes of X2 and 200 tonnes of X3, and this output will yield a contribution of £279 600. (Note that the signs in the final tableau are only reversed when additional scarce resources are obtained.)
(a) The company should invest in companies A, C and D since they yield positive NPVs. The company should be indifferent about investing in B since it yields a zero NPV.

(b) (i)

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>($)000</td>
<td>Present value</td>
<td>60</td>
<td>0</td>
<td>77</td>
</tr>
<tr>
<td>($)000</td>
<td>Investment at time 0</td>
<td>500</td>
<td>250</td>
<td>475</td>
</tr>
<tr>
<td>($)000</td>
<td>NPV per $1 invested at time 0</td>
<td>0.12</td>
<td>0</td>
<td>0.162</td>
</tr>
<tr>
<td>Ranking</td>
<td></td>
<td>2</td>
<td>4</td>
<td>1</td>
</tr>
</tbody>
</table>

The company should invest $475 000 in C and the balance of the funds ($225 000) in A.

(b) (ii) Other factors that should be considered are:
1. The risk of the cash flows of each company. A single cost of capital has been applied to all investments, but the riskier the investment the higher should be the cost of capital.
2. Are the cash flows of the companies highly correlated? If the cash flows for two of the companies are not highly correlated, the total risk of the two investments will be lower than the sum of the individual investments (see 'Portfolio analysis' in Chapter 12 for an explanation).
3. The experience of the staff in the individual companies and the extent to which they are dependent on a small number of key staff. Where the success of a company is dependent on a small number of key staff, obtaining the returns on the investment will be at risk should the staff leave.
4. The speed of the payback period if the cash flows can be used to reinvest in new projects. This is mainly applicable to when funds in future periods are restricted.

(c)

Let $X_1 =$ Proportion of funds invested in company A
Let $X_2 =$ Proportion of funds invested in company B
Let $X_3 =$ Proportion of funds invested in company C
Let $X_4 =$ Proportion of funds invested in company D

Maximize $60X_1 + 0X_2 + 77X_3 + 80X_4$

Subject to:

500$X_1 + 250X_2 + 475X_3 + 80X_4 + S_1 = 700$ (period 0 constraint)
75$X_1 + 30X_2 + 100X_3 + 150X_4 + S_2 = 89.6$ (period 1 constraint)
40$X_1 + 20X_2 + 30X_3 + 50X_4 + S_3 = 43.91$ (period 2 constraint)

$0 \leq X_j \leq 1$ (j = 1, … 3)

Note

1. The constraints are expressed in present values, but it is assumed that constraints applying will be expressed in period 1 and period 2 cash flows so period 1 = $80 (1.12)$ and period 2 = $35 (1.12)^2$.

The objective function specifies that NPV is to be maximized subject to the investment constraints for the three periods. The terms $S_1$, $S_2$, etc. represent the slack variables in the form of any unused funds for each period. The final term in the model indicates that any project (signified by $X_j$) cannot be undertaken more than once, but allows for a project to be partially undertaken. For a more detailed explanation of the meaning of the terms in a capital budgeting model you should refer to Appendix 26.1.

(d) The benefits from using linear programming in this situation are:
1. Complex investment problems and constraints can be modelled and solved using computer programs.
2 The model generates opportunity costs and marginal rates of substitution which can be useful for decision-making and control.

3 The model is flexible and can be applied to where fractional projects can be undertaken or modified to exclude fractional investments.

**Question 26.20**

(a) Let \(a, b, c, d, e, f, g\) represent the proportion of projects A, B, C, D, E, F and G accepted and \(X\) represent the amount of money placed on deposit in 2001 in £000. The NPV of £1000 placed on deposit is:

\[
\frac{\£1000(1.08)}{1.10} - £1000 = -£18 = £-0.018 \text{ (in £000)}
\]

Maximize \(Z = 39a + 35b + 5c + 15d + 14e + 32f + 24g - 0.018X\)

subject to

\[
\begin{align*}
80a + 70b + 55c + 60d + X + \text{DIV0} & \leq 250 \quad \text{(2001 constraint in £000)} \\
30a + 140c + 80f + 100g + \text{DIV1} & \leq 150 + 20b + 40c + 30d + 1.08X \\
\text{DIV0} & = 100 \\
\text{DIV1} & = 1.05 \text{ DIV0} \\
a, b, c, d, e, f, g & = 0 \\
a, b, c, d, e, f, g & \leq 1
\end{align*}
\]

Note that surplus funds are placed on deposit only for 2001. After 2003, capital is available without limit. Consequently, it is assumed at 2002 that it is unnecessary to maintain funds for future periods by placing funds on deposit to yield a negative NPV. It is assumed that capital constraints can be eased by project generated cash flows.

(b) The dual or shadow prices indicate the opportunity costs of the budget constraints for 2001 and 2002. The dual values indicate the NPV that can be gained from obtaining an additional £1 cash for investment in 2001 and 2002. The zero dual value for 2002 indicates that the availability of cash is not a binding constraint in 2002, whereas in 2001 the dual value indicates that £1 additional cash for investment in 2001 will yield an increase in total NPV of £0.25.

The dual values also indicate how much it is worth paying over and above the existing cost of funds. In this question it is worth paying up to £0.25 over and above the existing cost of capital for each £1 invested in 2001. In other words, the company should be prepared to pay up to 35% (10% acquisition cost + 25% opportunity cost) to raise additional finance in 2001.

Dual values can also be used to appraise any investments that might be suggested as substitutes for projects A to G. If the company identifies another project with a life of one year and a cash outflow of £130 000 at \(t_0\) and an inflow of £150 000 at \(t_1\), the NPV would be £3364 at a cost of capital of 10%. However, acceptance of the project would result in a reduction in NPVs from diverting funds from other projects of £32 500 (£130 000 \times £0.25). The project should therefore be rejected.

Dual values are not constant for an infinite range of resources. They apply only over a certain range. The question indicates that the values apply over a range between £120 000 and £180 000. Outside this range, it will be necessary to develop a revised model to ascertain the dual values that would be applicable. If resources continued to be increased, a point would be reached at which all potential projects with positive NPVs would have been accepted. Beyond this point, there would be no binding constraints and capital rationing would cease to exist.

(c) Capital rationing can be defined as a situation where there are insufficient funds available to undertake all those projects that yield a positive NPV. The literature distinguishes between hard and soft capital rationing. Hard capital rationing refers to those situations where firms do not have access to investment funds and
are therefore unable to raise any additional finance at any price. It is most unlikely that firms will be subject to hard capital rationing.

Soft capital rationing refers to those situations where capital rationing is internally imposed. For various reasons, top management may pursue a policy of limiting the amount of funds available for investment in any one period. Such policies may apply to firms that restrict their financing of new investments to internal funds. Alternatively, in a large divisionalized company, top management may limit the funds available to divisional managers for investment. Such restrictions on available funds may be for various reasons. For example, a company may impose its own restrictions on borrowing limits or avoid new equity issues because of a fear of outsiders gaining control of the business.

Where capital rationing exists, the NPV rule of accepting all positive NPV projects must be modified. Where capital is rationed for a single period, the approach outlined in Chapter 14 (see 'Capital rationing') should be applied. Where capital is rationed for more than one period, the optimal investment plan requires the use of mathematical programming (see Chapter 26). The application of the latter is based on a number of underlying assumptions that are unlikely to hold in the real world. For a discussion of these assumptions see the Appendix to Chapter 26.